System Engineering Report

Project: AXAF SIM/ GSE
Title: FAM coordinate conversion, alignment and calibration
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Abstract:

The Five-Axis Mount (FAM) is used at MSFC's X-ray Calibration Facility (XRCF) to move the SIM as though it were rigidly attached to the HRMA. The FAM is given an effective focal length and commanded to go to the HRMA's azimuth and elevation field angles; the FAM must calculate the three axis displacements of each of its three feet. The order may be reversed - i.e. given the feet displacements, the equivalent field angle (pair) may be calculated, as long as the actual feet displacements are close to the commanded feet displacements.

This SER details the mathematical procedure for mapping field angles into the three feet displacements and vv., given the three feet locations and taking into account non-orthogonal displacements in each of the three feet. Regrettably, non-orthogonalities (and installation alignments) matter, at least in the case of large movements in Y or Z that can cause defocus in X.

Simplified expressions are given that ignore non-orthogonalities. These expressions are useful at FAM checkout and acceptance test time. Given too are mathematics and procedures for measuring feet displacement non-orthogonalities. The procedures make use of an alignment cube mounted on the SIM simulator, two theodolites, a flat, and a folding mirror. While an interferometer provides some distinct advantages (and disadvantages), this setup can also be used at FAM acceptance test time to observe FAM resolutions, repeatabilities, stabilities and settling times.

Establishment of the FAM coordinate frame is described, making use of a 2" precision cube mounted at the SI origin on the SIM simulator. The optical installation and alignment of the FAM at XRCF is described, making use of a second alignment cube mounted on the FAM rear rail and parallel to the cube on the SIM simulator.

FAM encoder and motor gains are discussed.

Nomenclature:

Bold face is used to indicate vectors and matrices, e.g. **TAx** and **R54**. Non-bold face is used for scalars. M_{ij} denotes the matrix element on the i th row and j th column. V_i denotes the i th element of a vector. $[V_1,V_2,V_3,...]$ is a column vector, $[V_1,V_2,V_3,...]^T$ is a row vector. • is the matrix multiplication operator.

HRMA to focal plane geometry:

Figure 1 shows the HRMA and focal plane geometry.

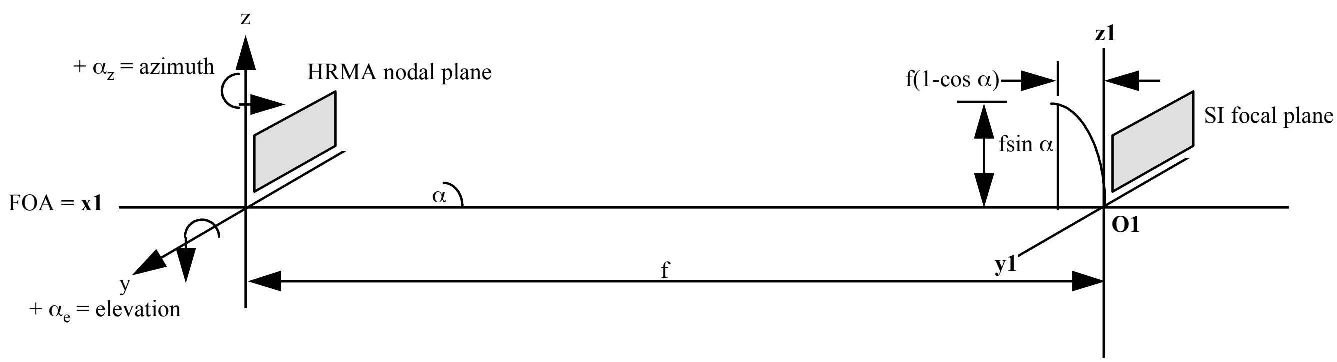

Figure 1. HRMA and focal plane geometry.

F1 (i.e. unit vectors **x1**,**y1**,**z1**) is coordinate frame #1, the SI focal plane. **F1**'s orientation is optically established by XRCF. Its origin **O1** is where HRMA images appear for sources on the Facility Optical Axis (FOA), i.e. on **x1**. f is the effective HRMA focal length, the radius of the spherical surface that off-axis images appear on. α_y is the HRMA field "elevation" angle, defined + for right-hand rotations about the +y axis, and α_z the HRMA field "azimuth" angle, defined + for right-hand rotations about the +z axis.

If $\boldsymbol{\psi}\mathbf{1} = [\psi 1_x, \psi 1_y, \psi 1_z]^T$ are the required SI rotations about **x1, y1** and **z1** respectively, and $\mathbf{S} = [S1_x, S1_y, S1_z]^T$ the required SI translations in the **x1**,**y1**,**z1** direction respectively (all angular and linear distances measured from their "boresight" position), then

Eq. 1a $\psi 1_z = \alpha_z$

Eq. 1b $\psi 1_y = \alpha_y$

Eq. 1c $\psi 1_x$ = any SI "roll" required at installation time only

Eq. 1c $S1_x = f\left[1 - \cos\sqrt{\alpha_z^2 + \alpha_y^2}\right]$

Eq. 1d $S1_y = -f \sin \alpha_z$

Eq. 1e $S1_z = f \sin \alpha_y$

ψ1 is the "SI angular coordinate", and **S1** the "SI linear coordinate". Since f is about 10m, and α_y and α_z about 0.5 degrees max, the max required values for $S1_x$,$S1_y$,$S1_z$ are about 0.8 mm, 90 mm, and 90 mm resp.

FAM stationary coordinate frame and non-orthogonality:

Figure 2 shows coordinate frame **F3** = [**x3**,**y3**,**z3]** , associated with the stationary structure of the FAM at boresight. Its origin is coincident with **F1**'s. **F3** is nominally parallel to **F1**, but need not be precisely so. **R32** depends on the attitude of an optical cube affixed to the FAM used to optically define **F3**, and on the installation attitude of the FAM itself on the optical bench at XRCF. **F3** is made by rotating **F1** by matrix **R32**. If **P1** is the coordinate of any point in **F1**, then its coordinate in **F3** is

Eq. 2 $\mathbf{P3} = \mathbf{R32} \bullet \mathbf{P1}$

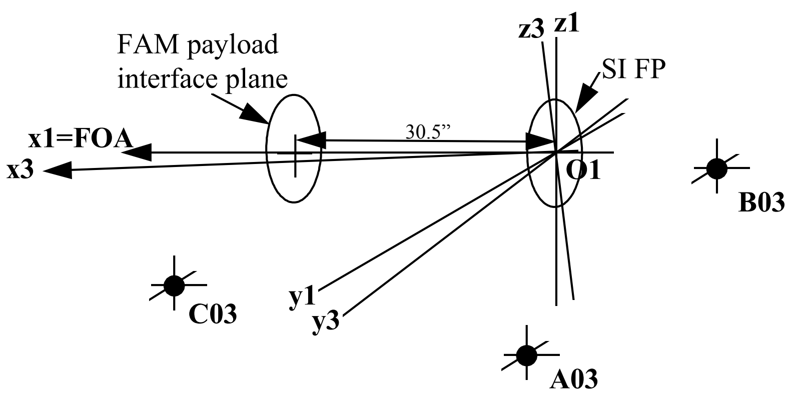

Figure 2. The FAM at boresight, its stationary coordinate frame **F3** = [**x3**,**y3**,**z3]**, and its three feet locations in **F3** - **A03**,**B03** and **C03**.

The intersection of the FAM payload nterface plane with the FOA, i.e. the center of the FAM aperture (about 66" above the table on which the FAM rests), is 28.5 (spec'd) + 2.0 (thickness of OBA simulator used to attach SIM to FAM) = 30.5 inches in the +**x1** direction from **O1**.

A "foot" defined to be the location of the 3-axis rotational pivot, assumed herein to be a single point, the coincident intersection of all three axes of rotation.

Figure 3 shows the directions a foot A,B or C moves when driven (or following) in x,y or z. All directions are measured in the FAM frame **F3**. Note that each foot can move in a slightly different direction in each of three axes. Each unit vector has three components, e.g. $\mathbf{TAx} = [TAx_x, TAx_y, TAx_z]^T$. All the vectors are close to their respective axes, so that $\mathbf{TAx} \cong [1, TAx_y, TAx_z]^T$,$\mathbf{TAy} \cong [\,TAy_x, 1, TAy_z]^T$, $\mathbf{TAz} \cong [\,TAz_x, TAz_y\,, 1]^T$, etc.

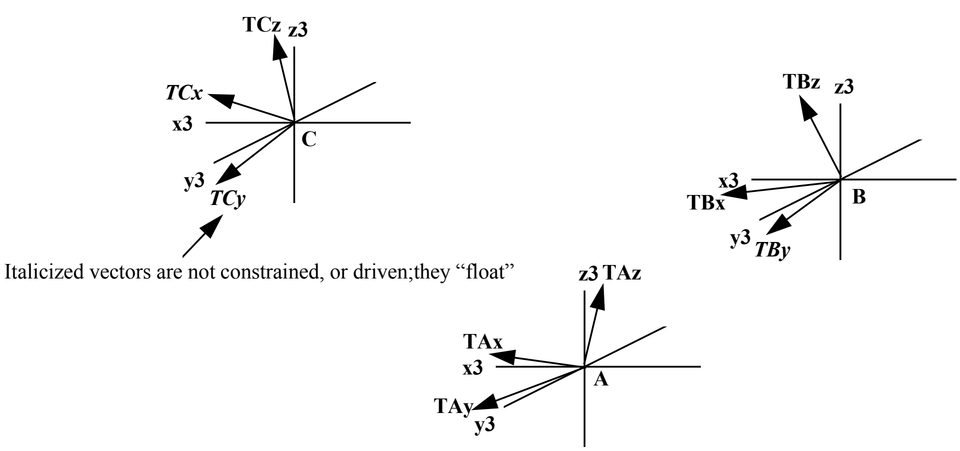

Figure 3. The translation vectors for the three feet A,B and C in each of three directions **x3**,**y3** and **z3**.

FAM and SI moving coordinate frames:

FAM frame **F4** is made by translating **F3**'s origin distance **v3**, as shown in Figure 4.

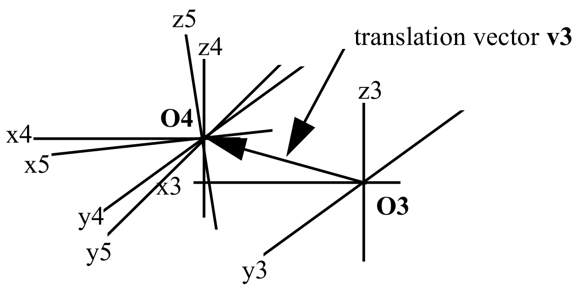

Figure 4. Translating and rotating the FAM.

FAM frame **F5** is made by rotating **F4** by matrix **R54**. If **P3** is the coordinate of any point in **F3**, then its coordinate in **F5** is

Eq. 3 $\quad \mathbf{P5} = \mathbf{R54} \bullet (\mathbf{P3}\text{-}\mathbf{v3})$.

Similarly, SI frame **F6** is made by translating **F1**'s origin distance **S1**, as shown in Figure 5.

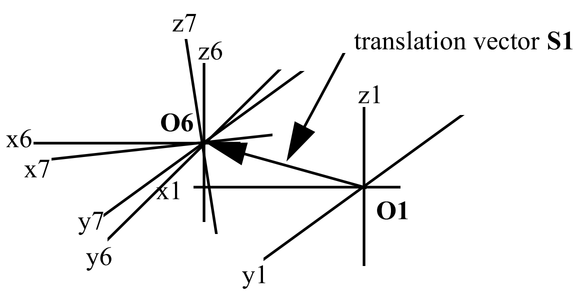

Figure 5. Translating and rotating the SIs.

SI frame **F7** is made by rotating **F6** by matrix **R76**. If **P1** is the coordinate of any point in **F1**, then its coordinate in **F7** is

Eq. 4 $\quad \mathbf{P7} = \mathbf{R76} \bullet (\mathbf{P1}\text{-}\mathbf{S1})$.

Since $\psi1_x, \psi1_y, \psi1_z$ are small, the order of rotation is unimportant, and the rotation matrix **R76** is, from Wertz, *Spacecraft Attitude Determination and Control,* pg. 764

Eq. 5a $$\mathbf{R76} = \mathbf{E321}(\psi1_z,\psi1_y,\psi1_x) \cong \begin{bmatrix} 1 & \psi1_z & -\psi1_y \\ -\psi1_z + \psi1_x\psi1_y & 1 + \psi1_x\psi1_y\psi1_z & \psi1_x \\ \psi1_x\psi1_z + \psi1_y & -\psi1_x + \psi1_z\psi1_y & 1 \end{bmatrix}$$

where $\psi1_x, \psi1_y, \psi1_z$ are in radians. Further, ignoring the cross terms due to small angles yields

Eq. 5b $$\mathbf{R76} = \mathbf{E321}(\psi1_z,\psi1_y,\psi1_x) \cong \begin{bmatrix} 1 & \psi1_z & -\psi1_y \\ -\psi1_z & 1 & \psi1_x \\ \psi1_y & -\psi1_x & 1 \end{bmatrix}$$

Note that since the SIs are rigidly tied to the FAM, **F7** has the same relationship to **F5** that **F1** does to **F3**. Thus if

Eq. 6a $\quad \mathbf{P3} = \mathbf{R32} \bullet \mathbf{P1}$ (this is Eq. 2), then also
Eq. 6b $\quad \mathbf{P5} = \mathbf{R32} \bullet \mathbf{P7}$.

Substituting from Eqs 2, 3 and 4 into Eq. 6b, and remembering that the inverse of any rotation matrix is simply its transpose,

Eq. 7 $\mathbf{P3} - \mathbf{v3} = \mathbf{R54}^T \cdot (\mathbf{Ra} \cdot \mathbf{P3} - \mathbf{Va})$, where
Eq. 7a $\mathbf{Ra} = \mathbf{R32} \cdot \mathbf{R76} \cdot \mathbf{R32}^T$ and
Eq. 7b $\mathbf{Va} = \mathbf{R32} \cdot \mathbf{R76} \cdot \mathbf{S1}$

Since Eq. 7 must be true for any and all **P3**s, choosing **P3** = **0** turns Eq. 7 into

Eq. 8a $\mathbf{v3} = \mathbf{R54}^T \cdot \mathbf{Va}$, and substituting this back into Eq. 7 yields
Eq. 8b $\mathbf{R54}^T \cdot \mathbf{Ra} \cdot \mathbf{P3} = \mathbf{P3}$

Since Eq. 8b must also be true for any and all **P3**s,

Eq. 9a $\mathbf{R54} = \mathbf{Ra} = \mathbf{R32} \cdot \mathbf{R76} \cdot \mathbf{R32}^T$; if **F3** is nearly parallel to **F1**
Eq. 9b $\mathbf{R54} \cong \mathbf{R76}$. Substituting Eq. 9a back into Eq. 8a yields
Eq. 9c $\mathbf{v3} = \mathbf{R32} \cdot \mathbf{S1}$

These two equations describe the rotation and translation the FAM frame **F3** must make to move the SIs to the required coordinates **R76** (i.e. $\boldsymbol{\psi}\mathbf{1}$) and **S1**.

Finding commanded feet displacements from a commanded HRMA angle:

Next we find the linear displacements the feet must make $\mathbf{\Delta A3}$,$\mathbf{\Delta B3}$,$\mathbf{\Delta C3}$ to accomplish to move the SIs to the required coordinates **R76** (i.e. $\boldsymbol{\psi}\mathbf{1}$) and **S1**. After the FAM rotates and translates from boresight, from Eq. 3 the new coordinate of foot A,B,C in frame **F3** is

Eq. 10a $\mathbf{A3} = \mathbf{R54}^T \cdot \mathbf{A03} + \mathbf{v3}$
Eq. 10b $\mathbf{B3} = \mathbf{R54}^T \cdot \mathbf{B03} + \mathbf{v3}$
Eq. 10c $\mathbf{C3} = \mathbf{R54}^T \cdot \mathbf{C03} + \mathbf{v3}$

The distance foot A,B,C moves from its boresight location in frame **F3** is

Eq. 11a $\mathbf{\Delta A3} = \mathbf{A3} - \mathbf{A03} = (\mathbf{R54}^T - \mathbf{I}) \cdot \mathbf{A03} + \mathbf{v3}$
Eq. 11b $\mathbf{\Delta B3} = \mathbf{B3} - \mathbf{B03} = (\mathbf{R54}^T - \mathbf{I}) \cdot \mathbf{B03} + \mathbf{v3}$
Eq. 11c $\mathbf{\Delta C3} = \mathbf{C3} - \mathbf{C03} = (\mathbf{R54}^T - \mathbf{I}) \cdot \mathbf{C03} + \mathbf{v3}$, where **I** is the 3x3 identity matrix.

But the feet actually move in non-orthogonal directions to **F3**. Thus

Eq. 12a $\mathbf{\Delta A3} = uAx \cdot \mathbf{TAx} + uAy \cdot \mathbf{TAy} + uAz \cdot \mathbf{TAz}$
Eq. 12b $\mathbf{\Delta B3} = uBx \cdot \mathbf{TBx} + uBy \cdot \mathbf{TBy} + uBz \cdot \mathbf{TBz}$
Eq. 12c $\mathbf{\Delta C3} = uCx \cdot \mathbf{TCx} + uCy \cdot \mathbf{TCy} + uCz \cdot \mathbf{TCz}$

where $\mathbf{uA} = [uAx,uAy,uAz]^T$ are the motor displacements for foot A along directions **TAx**,**TAy**,**TAz** resp., and similarly for feet B and C. TBx_y ,TBz_y ,TCx_y , TCy_z , TCz_x ,TCz_y may be set to zero, since they only direct motion into a floating direction.

Eq. 12 can be rewritten

Eq. 13a $\mathbf{\Delta A3} = \mathbf{TA} \cdot \mathbf{uA}$
Eq. 13b $\mathbf{\Delta B3} = \mathbf{TB} \cdot \mathbf{uB}$
Eq. 13c $\mathbf{\Delta C3} = \mathbf{TC} \cdot \mathbf{uC}$, where as good approximations

[1 TAy_x TAz_x]

Eq. 14a **TA** =
$$\begin{matrix} [TAx_y & 1 & TAz_y &] \\ [TAx_z & TAy_z & 1 &] \end{matrix}$$

Eq. 14b **TB** =
$$\begin{matrix} [1 & TBy_x & TBz_x &] \\ [0 & 1 & 0 &] \\ [TBx_z & TBy_z & 1 &] \end{matrix}$$

Eq. 14c **TC** =
$$\begin{matrix} [1 & 0 & 0 &] \\ [0 & 1 & 0 &] \\ [TCx_z & TCy_z & 1 &] \end{matrix}$$

The motor displacements **uA**,**uB**,**uC** can be found with

Eq. 15a $\mathbf{uA} = \mathbf{TA}^{-1} \bullet \mathbf{\Delta A3}$

Eq. 15b $\mathbf{uB} = \mathbf{TB}^{-1} \bullet \mathbf{\Delta B3}$

Eq. 15c $\mathbf{uC} = \mathbf{TC}^{-1} \bullet \mathbf{\Delta C3}$

$\mathbf{TA}^{-1}$, $\mathbf{TB}^{-1}$, and $\mathbf{TC}^{-1}$ need only be found once, a result of non-orthogonality measurements. As a good approximation,

Eq. 16a $\mathbf{TA}^{-1}$ =
$$\begin{matrix} [1 & -TAy_x & -TAz_x &] \\ [-TAx_y & 1 & -TAz_y &] \\ [-TAx_z & -TAy_z & 1 &] \end{matrix}$$

Eq. 16b $\mathbf{TB}^{-1}$ =
$$\begin{matrix} [1 & -TBy_x & -TBz_x &] \\ [0 & 1 & 0 &] \\ [-TBx_z & -TBy_z & 1 &] \end{matrix}$$

Eq. 16c $\mathbf{TC}^{-1}$ =
$$\begin{matrix} [1 & 0 & 0 &] \\ [0 & 1 & 0 &] \\ [-TCx_z & -TCy_z & 1 &] \end{matrix}$$

Figure 6 shows the calculation sequence.

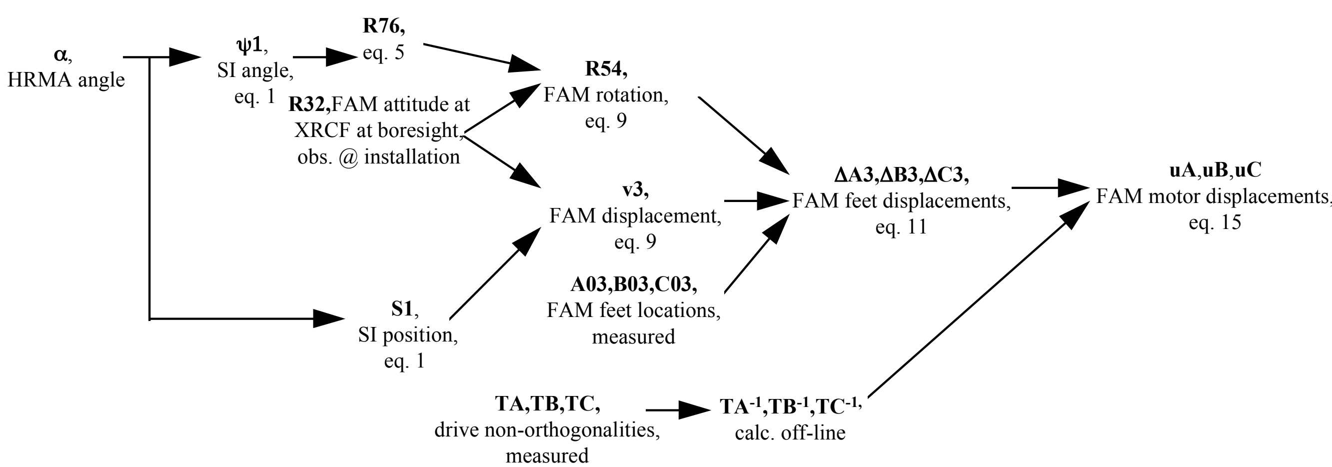

Figure 6. The sequence of calculations for finding commanded feet displacements from a commanded HRMA angle.

Simplified expressions for finding commanded feet displacements, ignoring XRCF, with no non-orthogonalities:

Set **R32** = **I**; this aligns the FAM to XRCF, or in effect ignores the prescence of XRCF. This is useful for FAM tests and calibrations that are conducted externally to XRCF. Further, set **TA** = **TB** = **TC** = **I** ; this declares "no non-orthogonalities", useful at FAM acceptance test time where non-orthogonalities are not inspected. Then Eq. 13 becomes

Eq. 24a $\mathbf{uA} = (\mathbf{R76}^T - \mathbf{I}) \bullet \mathbf{A03} + \mathbf{S1}$

Eq. 24b $\mathbf{uB} = (\mathbf{R76}^T - \mathbf{I}) \bullet \mathbf{B03} + \mathbf{S1}$

Eq. 24c $\mathbf{uC} = (\mathbf{R76}^T - \mathbf{I}) \bullet \mathbf{C03} + \mathbf{S1}$

Simplify the nomenclature by defining azimuth angle $a = \psi1_z$, elevation angle $e = \psi1_y$, and roll angle $r = \psi1_x$, defocus $v_x = S1_x$, horizontal translation $v_y = S1_y$, and vertical translation $v_z = S1_z$. Also drop the appendage "3" to denote **F3 and** "0" to denote nominal position, so that **A**,**B**,**C** = **A03**,**B03**,**C03**. Then from Eq. 5b

Eq. 25 $$\mathbf{R76}^T - \mathbf{I} = \begin{bmatrix} 0 & -a & e \\ a & 0 & -r \\ -e & r & 0 \end{bmatrix}$$

and dropping the uB_y , uC_x , uC_y terms since they "float",

Eq. 26a $uA_x = -a \bullet A_y + e \bullet A_z + v_x$

Eq. 26b $uA_y = a \bullet A_x - r \bullet A_z + v_y$

Eq. 26c $uA_z = -e \bullet A_x + r \bullet A_y + v_z$

Eq. 26d $uB_x = -a \bullet B_y + e \bullet B_z + v_x$

Eq. 26e $uB_z = -e \bullet B_x + r \bullet B_y + v_z$

Eq. 26f $uC_z = -e \bullet C_x + r \bullet C_y + v_z$

where all angles are in radians.

Finding actual HRMA angles from actual feet displacements:

This procedure assumes that actual feet positions are close to commanded feet positions. It uses an incremental technique that permits one to ignore non-orthogonalities and coordinate frame misalignments. Thus the simplified equations developed above may be used.

Eq. 26 can be restated in matrix form

Eq. 27a $$\begin{bmatrix} uA_x \\ uA_y \\ uA_z \\ uB_x \\ uB_z \\ uC_z \end{bmatrix} = \begin{bmatrix} -A_y & A_z & 0 & 1 & 0 & 0 \\ A_x & 0 & -A_z & 0 & 1 & 0 \\ 0 & -A_x & A_y & 0 & 0 & 1 \\ -B_y & B_z & 0 & 1 & 0 & 0 \\ 0 & -B_x & B_y & 0 & 0 & 1 \\ 0 & -C_x & C_y & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ e \\ r \\ v_x \\ v_y \\ v_z \end{bmatrix}$$, or we define

Eq. 27b $\mathbf{u6} = \mathbf{M66} \bullet \mathbf{s6}$. Then

Eq. 28 $\mathbf{s6} = \mathbf{M66}^{-1} \bullet \mathbf{u6}$

The actual SI location $S1_x$, $S1_y$, $S1_z$ is the commanded $S1_x$, $S1_y$, $S1_z$ plus the correction $s6_4$, $s6_5$, $s6_6 = v_x$, v_y , v_z obtained from Eq. 28, when **M66** represents the nominal (i.e. boresight or home) feet positions, and **u6** represents the *difference* between the actual feet positions and the commanded feet positions. $\mathbf{M66}^{-1}$ need only be computed once. Small changes in SI angles $\psi1_x$, $\psi1_y$, $\psi1_z$ = $s6_1$, $s6_2$, $s6_3$ = a,e,r can be ignored from the HRMA's point of view.

As an example, suppose **A** = **A03** = $[-1,1,-1]^T$, **B** = **B03** = $[-1,-1,-1]^T$, **C** = **C03** = $[1,0,-1]^T$. Then $\mathbf{M66}^{-1}$ is

$$
\begin{array}{c c}
 & \begin{array}{cccccc} uA_x & uA_y & uA_z & uB_x & uB_z & uC_x \end{array} \\
\begin{array}{c} a \\ e \\ r \\ v_x \\ v_y \\ v_z \end{array} &
\left[\begin{array}{cccccc}
-0.50 & 0.00 & 0.00 & 0.50 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.25 & 0.00 & 0.25 & -0.50 \\
0.00 & 0.00 & 0.50 & 0.00 & -0.50 & 0.00 \\
0.50 & 0.00 & 0.25 & 0.50 & 0.25 & -0.50 \\
-0.50 & 1.0 & -0.50 & 0.50 & 0.50 & 0.00 \\
0.00 & 0.00 & 0.25 & 0.00 & 0.25 & 0.50
\end{array}\right]
\end{array}
$$

Finally, find the equivalent HRMA angles α from Eq. 1d and e and the corrected **S1**

Eq. 30a $\alpha_z = \sin^{-1}(-S1_y / f)$

Eq. 30b $\alpha_y = \sin^{-1}(S1_z / f)$

Figure 7 shows the sequence of calculations.

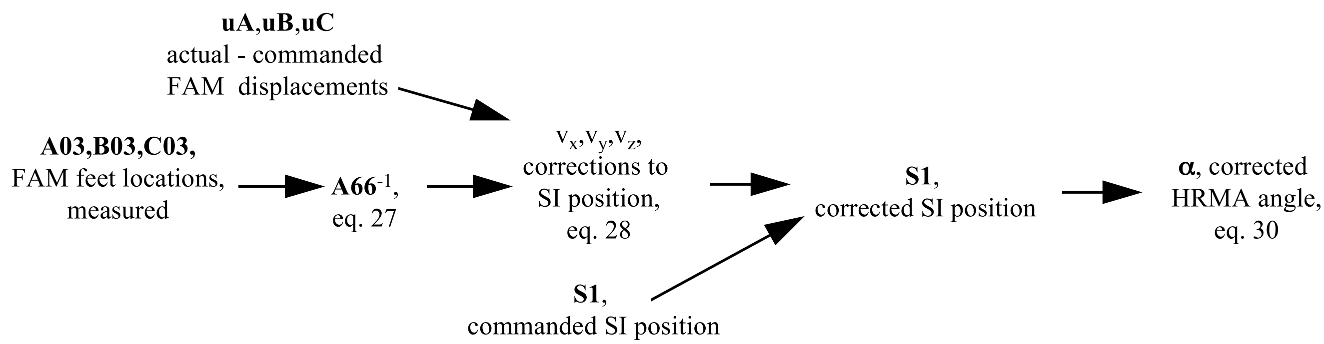

Figure 7. The sequence of calculations for finding actual HRMA angles from actual feet displacements.

Mathematics of measuring FAM non-orthogonalities:

All the 12 critical components of **TA**, **TB** and **TC** may be found by simultaneously observing the rotation (a,e,r) and translation (v_x,v_y,v_z) of the alignment cube on the SIM simulator at the FAM origin in response to translations of the three FAM feet together in the three directions x, y and z.

Starting at home, first the A_y actuator is moved a large distance $uA_y/2$ in the -y direction, and the three angular (a1,e1,r1) and three linear ($v1_x$,$v1_y$,$v1_z$) coordinates of the cube noted. Then the A_y actuator is moved distance uA_y in the +y direction, and the final three angular (a2,e2,r2) and three linear ($v2_x$,$v2_y$,$v2_z$) coordinates of the cube noted. Using the definitions of Eq. 27, the net cube movement is $\mathbf{s6} = [a,e,r,v_x,v_y,v_z]^T = [a2,e2,r2,v2_x,v2_y,v2_z]^T - [a1,e1,r1,v1_x,v1_y,v1_z]^T$. The FAM essentially only translates - any rotation is due to non-parallelism of rails between the feet.

From Eq. 13, feet A,B,C move distance

Eq. 51a $\Delta A_x = TAy_x \bullet uA_y$

Eq. 51b $\Delta A_y = TAy_y \bullet uA_y$

Eq. 51c $\Delta A_z = TAy_z \bullet uA_y$
Eq. 51d $\Delta A_x = TBy_x \bullet uA_y$
Eq. 51e $\Delta B_z = TBy_z \bullet uA_y$
Eq. 51f $\Delta C_z = TCy_z \bullet uA_y$

Some of these relationships are approximations that ignore second order terms arising from slightly different translation vectors between a pair of feet. Define

Eq. 52 $\mathbf{\Delta ABC6} = [\Delta A_x , \Delta A_y , \Delta A_z , \Delta B_x , \Delta B_z , \Delta C_z]^T$

Eq. 27b can be modified to include non-orthogonalities to read

Eq. 53 $\mathbf{\Delta ABC6} = \mathbf{M66} \bullet \mathbf{s6}$

Thus by observing **s6**, **ΔABC6** can be found, and since uA_y is known, the set TAy_x, TAy_y, TAy_z, TBy_x, TBy_z, TCy_z may be found from Eq. 51. As a check, TAy_y should be near unity.

This procedure for translating y motion is repeated a second time, but in the x direction. Both A and B feet must be moved nominally the same distance $uA_x = uB_x$. Feet A,B,C move distance

Eq. 54a $\Delta A_x = TAx_x \bullet uA_x$
Eq. 54b $\Delta A_y = TAx_y \bullet uA_x$
Eq. 54c $\Delta A_z = TAx_z \bullet uA_x$
Eq. 54d $\Delta A_x = TBx_x \bullet uA_x$
Eq. 54e $\Delta B_z = TBx_z \bullet uA_x$
Eq. 54f $\Delta C_z = TCx_z \bullet uA_x$

A second **s6** is observed, a second **ΔABC6** calculated, and the set TAx_x, TAx_y, TAx_z, TBx_x, TBx_z, TCx_z may be found from Eq. 54. As a check, TAx_x and TBx_x should be near unity.

Finally, this procedure is repeated a third time for translating z motion. All feet, A, B and C, must be moved nominally the same distance $uA_z = uB_z = uC_z$. Feet A,B,C move distance

Eq. 55a $\Delta A_x = TAz_x \bullet uA_z$
Eq. 55b $\Delta A_y = TAz_y \bullet uA_z$
Eq. 55c $\Delta A_z = TAz_z \bullet uA_z$
Eq. 55d $\Delta A_x = TBz_x \bullet uA_z$
Eq. 55e $\Delta B_z = TBz_z \bullet uA_z$
Eq. 55f $\Delta C_z = TCz_z \bullet uA_z$

A third **s6** is observed, a third **ΔABC6** calculated, and the set TAz_x, TAz_y, TAz_z, TBz_x, TBz_z, TCz_z may be found from Eq. 55. As a check, TAz_z , TBz_z and TCz_z should be near unity.

Measuring stage encoder and motor gains:

There are six translation stage encoders, one each for the Ax, Ay, Az, Bx, Bz and Cz actuators. They are considered to measure motion along the **TAx**,**TAy**,**TAz**,**TBx**,**TBz**,**TCz** directions, i.e. along the actual directions of motion, not the ideal ones. They must be installed parallel (< 0.5 °, TBR) to the stage rails, so that any cosθ reduction (< 40 ppm, TBR) in apparent gain can be ignored. If this criteria cannot be met, the apparent gains must be measured (along the actual directions of motion) , a subject not further discussed herein.

The encoders are glass scale devices that have nominally the same calibrations: a published gain K0e µm/counts at a specified temperature T0, and a published temperature coefficient dKedT in µm/counts-degC . At XRCF time, all the encoders are at about the same temperature T (near 10 °C) , so that all the encoders will have the same gain Ke = Ke0 + (T-T0) • dKedT. However, in the interest of generality , a separate Ke is kept for each encoder.

There are six translation stage motors, one each for the Ax, Ay, Az, Bx, Bz and Cz actuators. They are considered to drive motion along the **TAx**,**TAy**,**TAz**,**TBx**,**TBz**,**TCz** directions, i.e. along the actual directions of motion. They are stepping motor (and worm gear, in the z direction) and lead screw arrangements. Their gains Km µm/microstep can be found from published specifications for the motion train components. A separate gain Km is kept, in general, for each motor.

Certainly, Ke and Km must agree. This amounts to calibrating Ke against Km if there is more confidence in lead screw pitch. If there is more confidence in encoder alignment, it amounts to calibrating Km against Ke. This subject is not further addressed herein.

Establishing the FAM coordinate frame F3:

Figure 8 shows an alignment cube with its center at the SI origin. The 2" cube has all faces orthogonal within ±10 arcsec (TBR). It is mounted on the ISIM simulator, in turn mounted on the OBA simulator, in turn mounted to the FAM payload interface. With the FAM at home, the faces are nominally parallel to directions of motion at the feet. If the feet motion directions are in turn nominally parallel to the XRCF coordinate frame, the cube's face will be nominally parallel to **F1**. Ideally, a clear aperture will be visible on each of the six cube faces from outside the FAM structure.

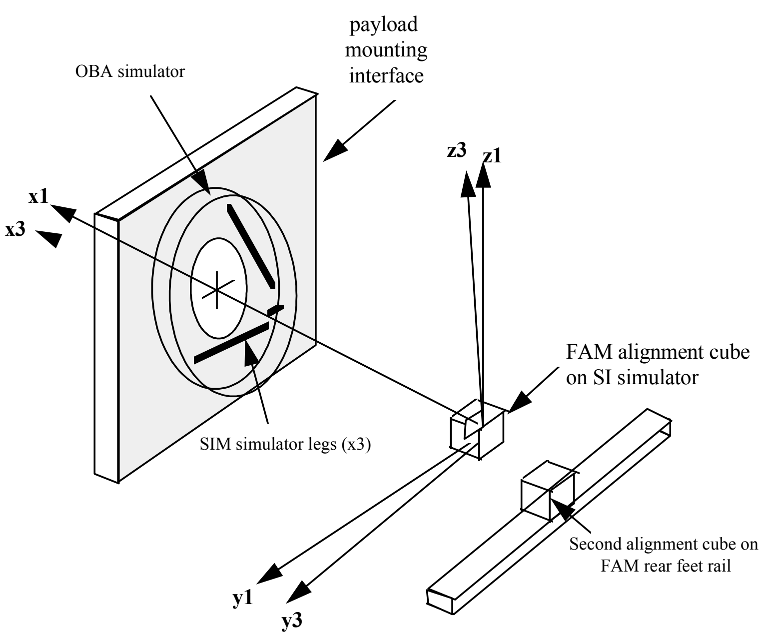

Figure 8. The FAM alignment cube at the SI origin, and a second cube on the rear foot rail.

Since the SIM itself, rather than the SIM simulator, is mounted on the FAM at the time the FAM is installed at XRCF, a second cube (or set of polished surfaces) is required on the stationary FAM structure. The rail that connects the two rear feet is the best candidate. The faces of the second cube are nominally parallel to the primary cube; both must be surveyed to establish an attitude between them.

Installing and aligning the FAM at XRCF:

Nominally, the FAM is installed with its frame **F3** parallel to XRCF's frame **F1**. If the SIM simulator were mounted on the FAM at the time the FAM is installed at XRCF, Figure 9 shows the technique that would be used. At a minimum, two angles, ϕ (azimuth) and θ (elevation), are to be observed with a theodolite viewing the +x face of the alignment cube, and the roll angle ψ assumed zero. If accuracy requires, a second theodolite observation of the alignment cube's +y face returns the roll angle ψ. The figure shows positive ϕ,θ and ψ.

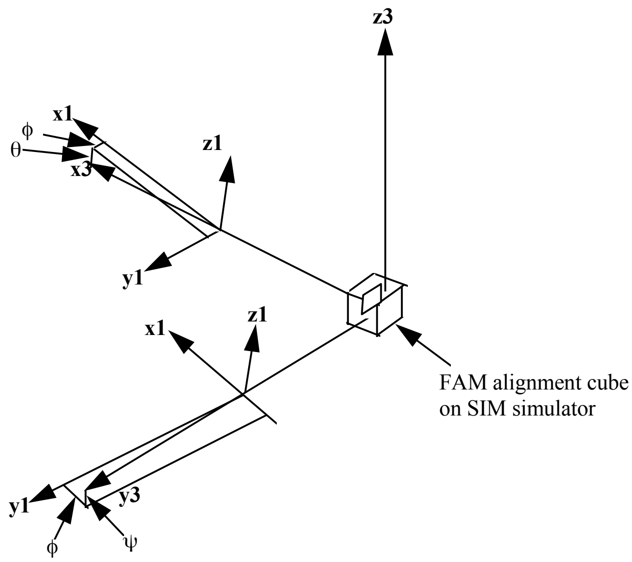

Figure 9. Measuring FAM attitude with respect to XRCF, if SM simulator could be mounted on the FAM in the IC.

Since ϕ,θ,ψ are small, the order of rotation is unimportant, and the rotation matrix **R32** is, from Wertz, *ibid,* pg. 764

Eq. 61a $\mathbf{R32} = \mathbf{E321}(\phi,\theta,\psi) \cong \begin{bmatrix} 1 & \phi & -\theta \\ -\phi + \psi\theta & 1 + \psi\theta\phi & \psi \\ \psi\phi + \theta & -\psi + \theta\phi & 1 \end{bmatrix}$, and further

Eq. 61b $\mathbf{R32} \cong \begin{bmatrix} 1 & \phi & -\theta \\ -\phi & 1 & \psi \\ \theta & -\psi & 1 \end{bmatrix}$

where ϕ,θ,ψ are in radians. In practice, since the SIM itself is on the FAM at installation time, ϕ,θ,ψ must be measured using the alignment cube mounted on the FAM rear rail, and corrections made for the tilt of the SIM simulator cube with respect to the cube on the rail. If ϕ',θ',ψ' are the rear rail cube angles with respect to **F1**, and $\delta\phi,\delta\theta,\delta\psi$ the angles between of the SIM simulator cube with respect to the rear rail cube, then $\phi,\theta,\psi = \phi',\theta',\psi' + \delta\phi,\delta\theta,\delta\psi$.

Setup and procedure for measuring non-orthogonalities:

Non-orthogonalities **TA**,**TB** and **TC** are measured with aid of two theodolites, a flat, and a folding mirror. Three setups are required, one for y, x and z motion, as shown in Figure 10.

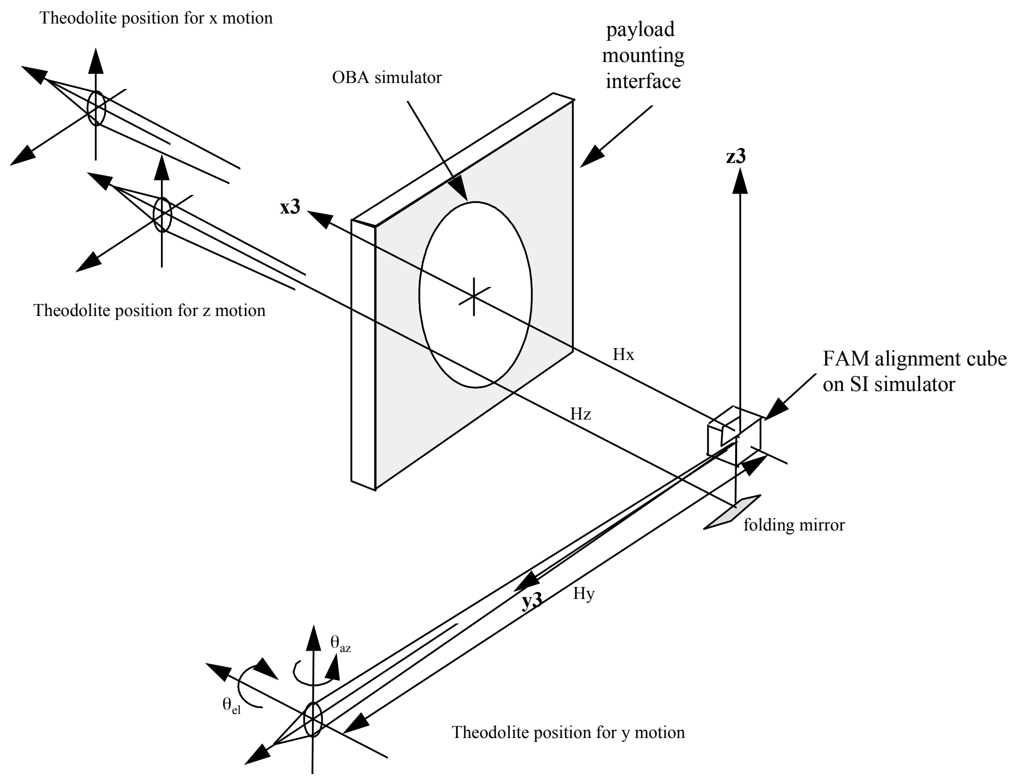

Figure 10. Theodolite setups for measuring non-orthogonalities **TA**,**TB** and **TC** .

Consider first the setup for measuring y motion non-orthogonalities. Primary theodolite Ty is placed with its standing axis (its vertical axis) placed distance H_y down the **y3** axis from the *center* of the alignment cube on the SI simulator, with the FAM at home. H_y should be much larger than the 10" range of possible y motion uA_y . Secondary theodolite Tx is similarly placed distance H_x down the x axix from the cube. Tx is used only to find the change in the cube's elevation angle, all other variables are found with Ty.

The alignment cube has a reticle engraved on all six faces. We wish to estimate the position of (i.e. point to and focus on) the cube's center, but can only observe its faces. When estimating the cube's (linear) position, view the reticle position on both the +y and -y faces, and sight its midpoint. Due to refraction and the fact the face does rotate slightly during travel, the apparent positon of the -y face will be slightly in error, leading to a slight error in estimating position of the cube's center, which is ignored.

Ty's and Tx's axes are nominally parallel to **F3**, with the FAM at home, but need not be precisely so. With the FAM at home, position Ty so that simultaneously the +y face of the alignment cube is normal to the its line-of-sight (LOS) (i.e. autocollimate off the +y face), and the cube center is near the center of the field-of-view (FOV). This requires both translating and rotating Ty.

As described in **Mathematics of measuring FAM non-orthogonalities**, the FAM is first translated distance $uA_y/2$ (as much as 5") in the -y direction. Note Ty angles θy_{ax} and θy_{el} (using the sign convention in Figure 10, not Ty's sign convention) required to autocollimate the +y face; call them $\phi y1_{ax}$ and $\phi y1_{el}$. Then sight the cube center, note Ty angles θy_{ax} and θy_{el} ; and call them $\theta y1_{ax}$ and $\theta y1_{el}$. Position Tx so that it can autocollimate off the +x face of the cube, and record its elevation angle $\phi x1_{el}$ (using the sign convention in Figure 10, not Tx's sign convention) .

Next, move distance uA_y (as much as 10") in the +y direction. Use a flat to remember Tx's elevation angle, and move Tx in the +y direction where it can once again autocollimate off the cube's +x face. Again find the cube's angles $\phi y2_{ax}$, $\phi y2_{el}$, and $\phi x2_{el}$, and its position $\theta y2_{ax}$ and $\theta y2_{el}$.

The change in cube angles $\Delta y\phi_{ax}$, $\Delta\phi x_{el}$, $\Delta\phi y_{el}$ = $\phi y2_{ax}$, $\phi x2_{el}$, $\phi y2_{el}$ - $\phi y1_{ax}$, $\phi x1_{el}$, $\phi y1_{el}$ are the angles a,e,r of Eqs. 25, 26 and 27. The change in cube position $\Delta\theta y_{ax}$, $\Delta\theta y_{el}$ = $\theta y2_{ax}$, $\theta y2_{el}$ - $\theta y1_{ax}$, $\theta y1_{el}$ permit the determination of FAM translation **v** as follows, where the coordinate transformation algebra is omitted.

Eq. 71
$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \cong \begin{bmatrix} \Delta\theta y_{ax} (H_y - uA_y) \\ uA_y \\ -\Delta\theta y_{el} (H_y - uA_y) \end{bmatrix}$$

Note that since the actual displacement v_y is not observed (it is only approximated with uA_y), TAy_y is not observed, and must be approximated as unity (It could be observed with additional work, using theodolite Tx).

This procedure is repeated a second time for x axis motion; Tx becomes the primary theodolite, and Ty the secondary. Finally this procedure is repeated a third time for z axis motion; Tz is the primary theodolite, and Ty the secondary.