Lynx Telescope Mirror Physics for Dummies

Bandwidth, Area and Resolution

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June 7, 2017

Study Goals and Assumptions

Goals:

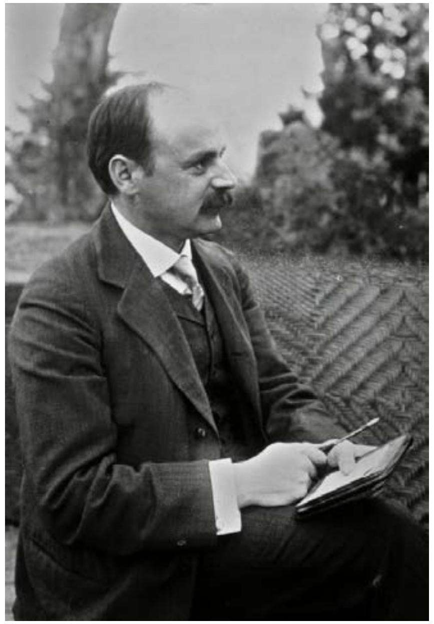
- Develop a simple parametric model of telescope bandwidth, collecting area and resolution
- Use model to explore performance trade-offs vs. mirror design parameters
- Provide the model to the community via an Excel-based tool

Assumptions:

- Single Wolter-Schwarzschild mirror P+S pair (not nested)
- Idealized mirrors (perfect figure, zero assembly errors, etc.)
- Simple iridium coating
- Detectors perfectly curved to match focal surface

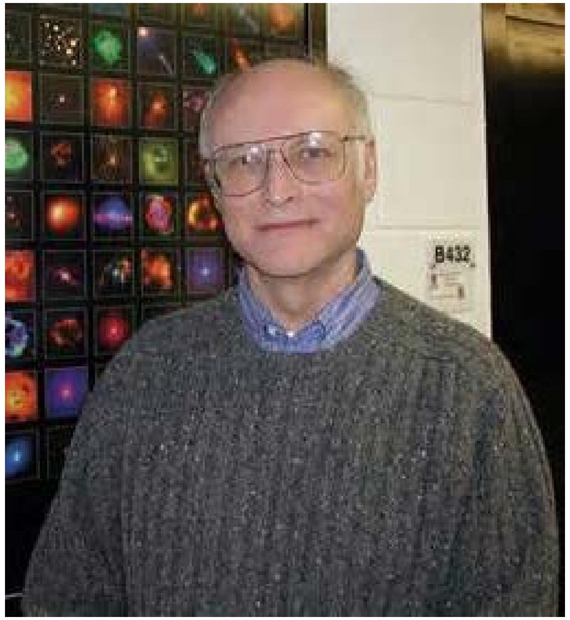
Acknowledgements

"If I have seen further, it is by standing on the shoulders of giants" (Isaac Newton, 1676)



Karl Schwarzschild
(1873–1916)

Hans Wolter
(1911–1978)

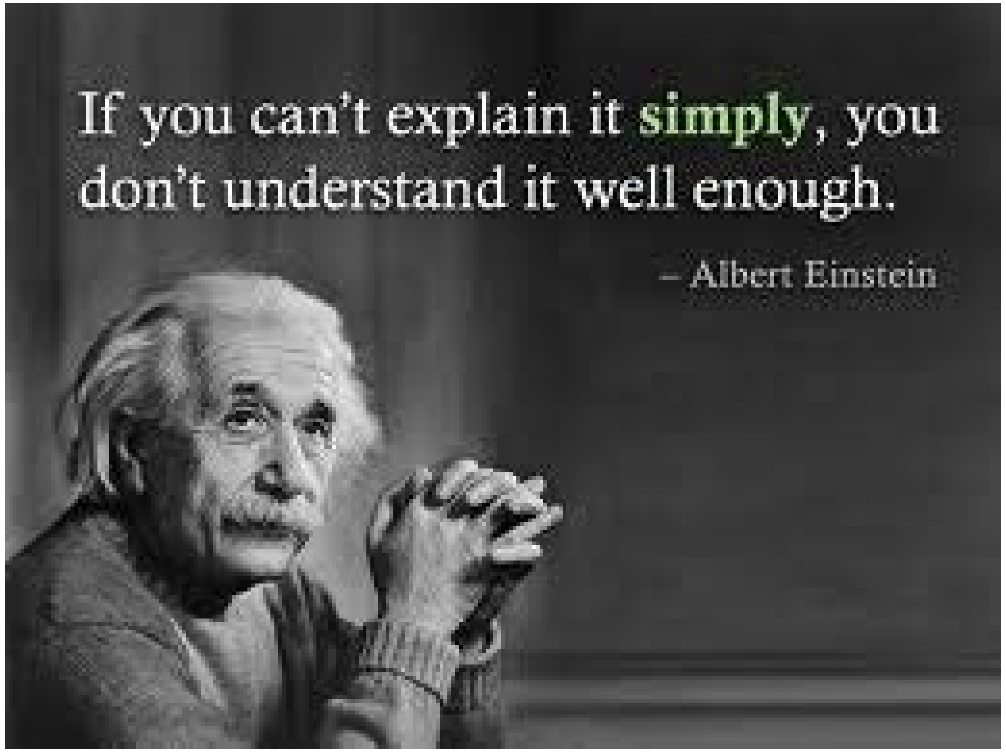


Leon P. Van Speybroeck
(1935 – 2002)

… and many useful papers & discussions from Dan Schwartz, Ron Elsner, Timo Saha, Paul Reid, Lester Cohen, James Harvey, Bernd Aschenbach, Ryan Allured, and many others I'm sure that I have forgotten …

Why Study Just a Single Mirror Shell?

- A great deal of insight can be obtained by studying a single mirror shell
- Mirror bandwidth and resolution are strongly determined by outermost mirror
- Mirror area is strongly determined by outermost mirror (shell area goes like square of radius).



Step 1: Telescope Energy Bandwidth

Telescope Bandwidth

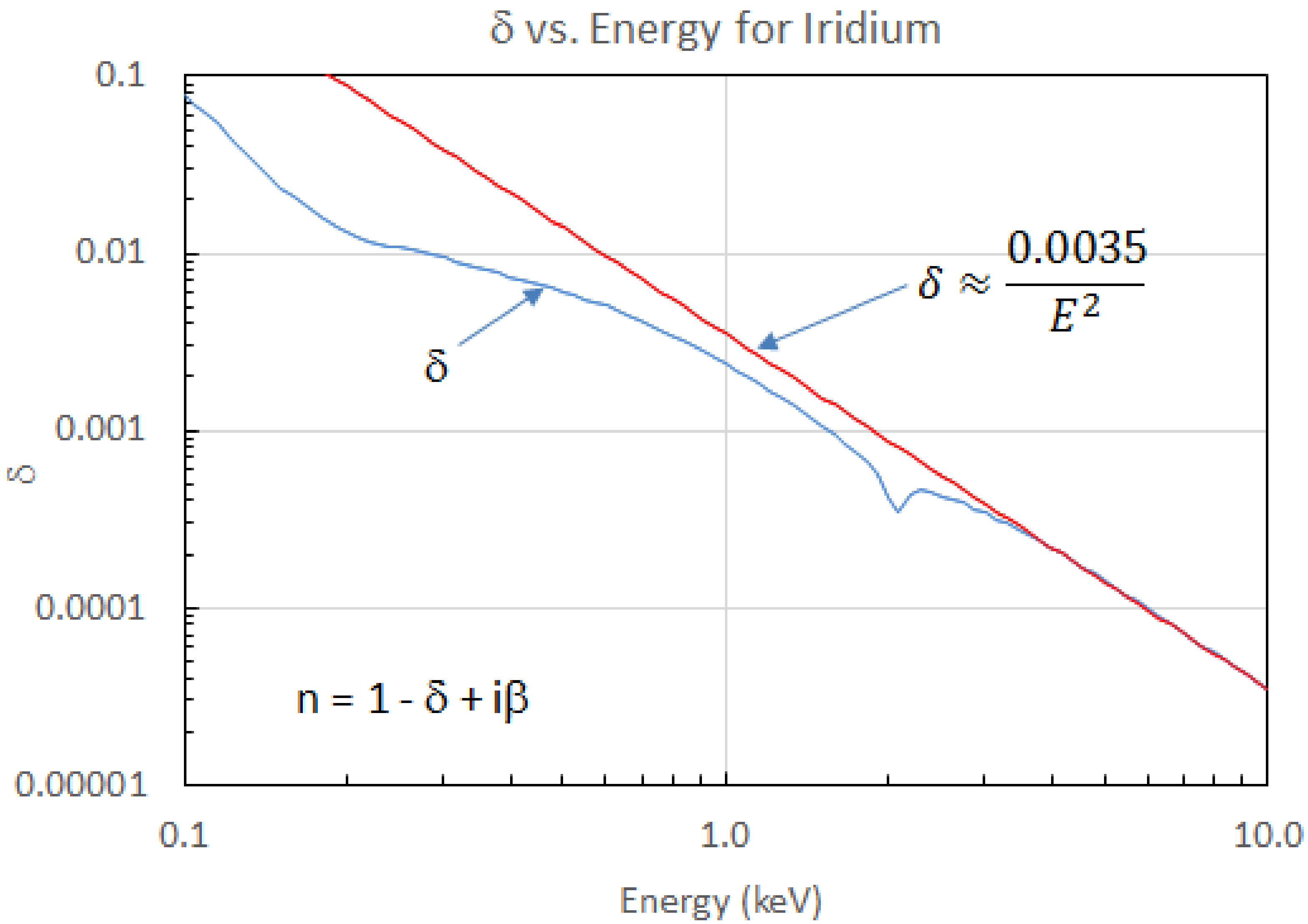
- Telescope bandwidth is strongly determined by the cutoff energy of outermost mirror
- Cutoff energy is determined by the critical angle of x-ray reflection:

$$\alpha_c = \sqrt{2\delta(E)}$$

where δ is the real part of the mirror coating's complex refractive index $n = 1 - \delta + i\beta$

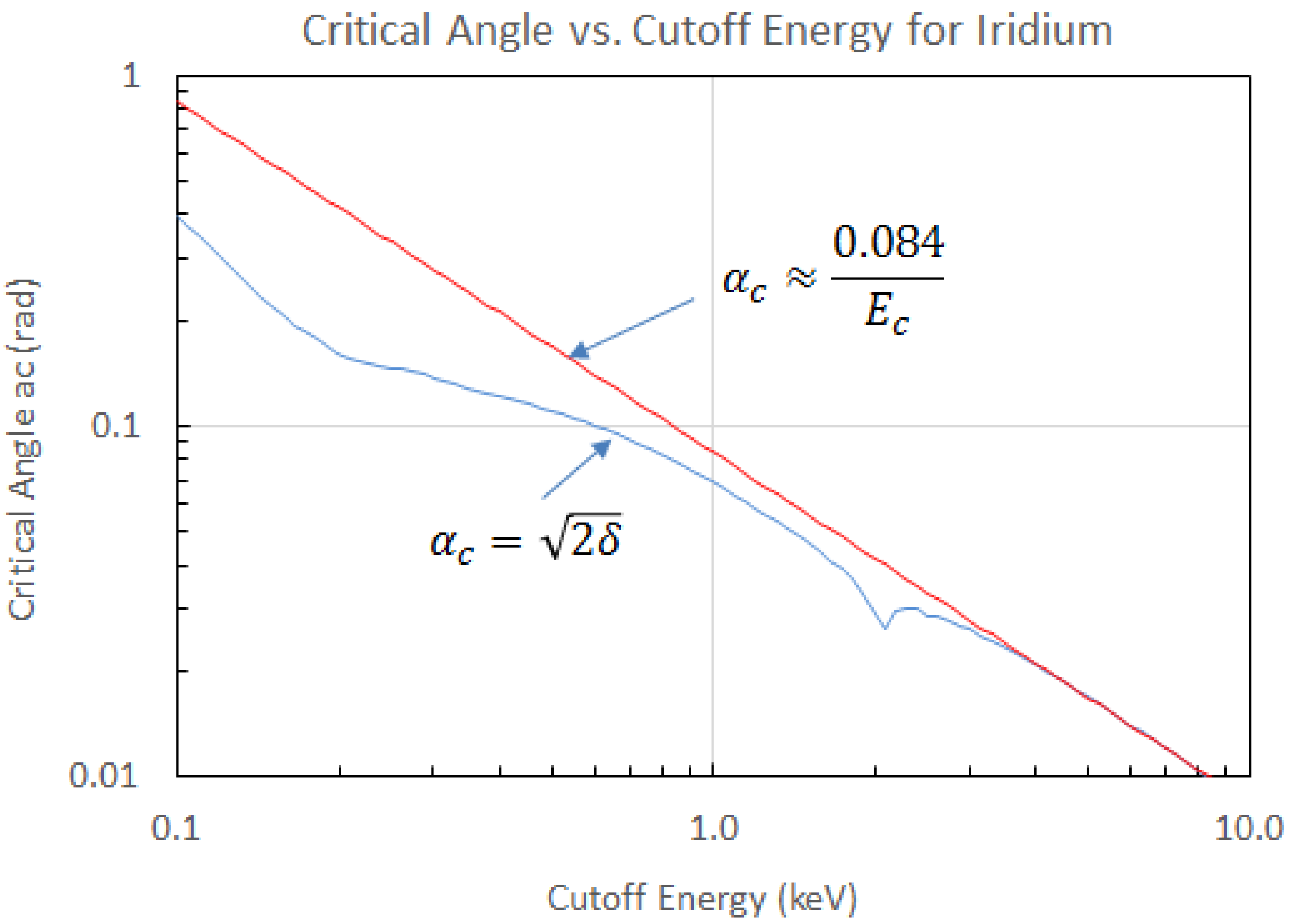
- Mirrors with graze angle above the critical angle reflect x rays with energies above the critical energy very poorly

Atomic Physics Scales δ like $1/E^2$

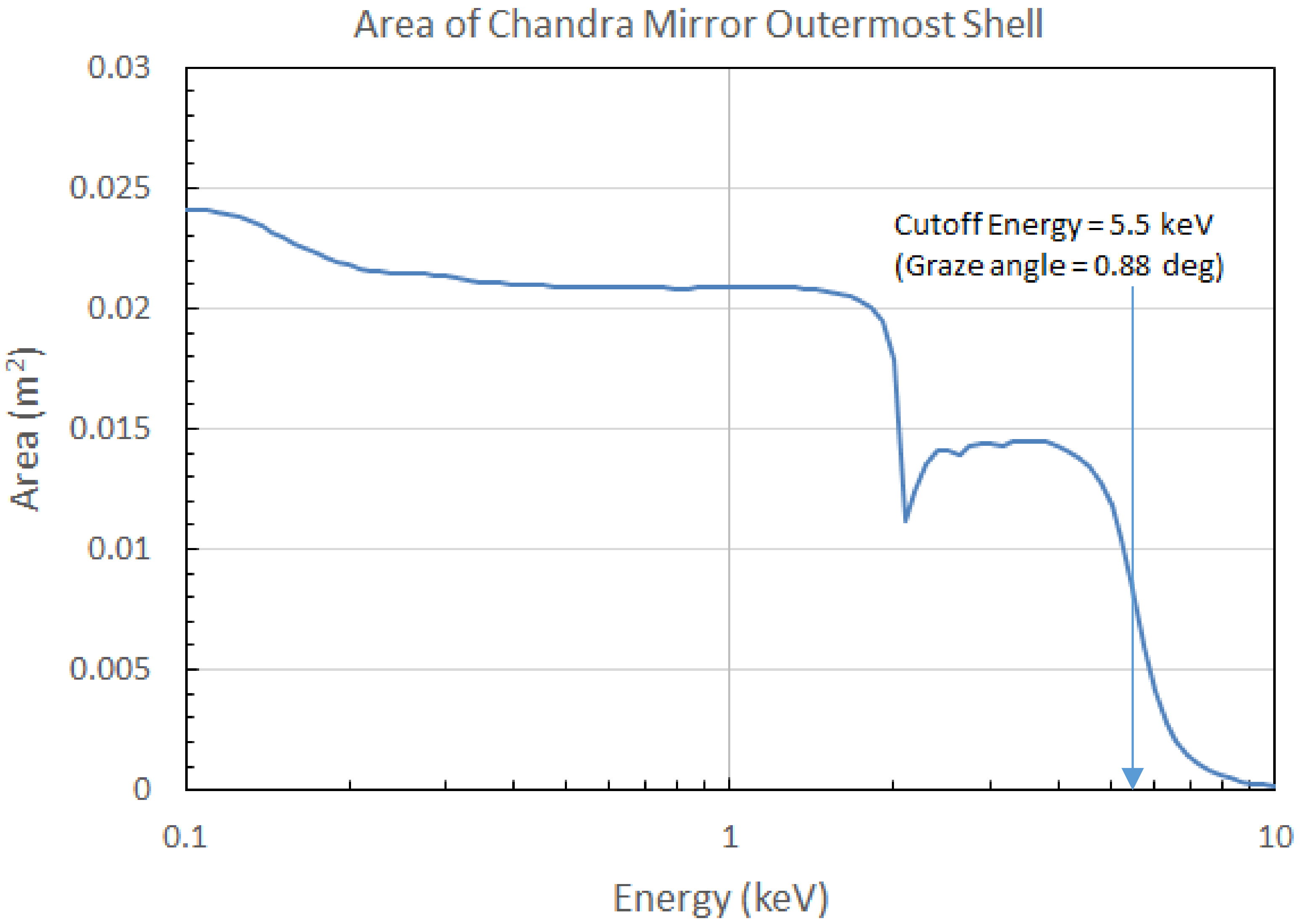


Mirror Critical Angle Scales like 1/E

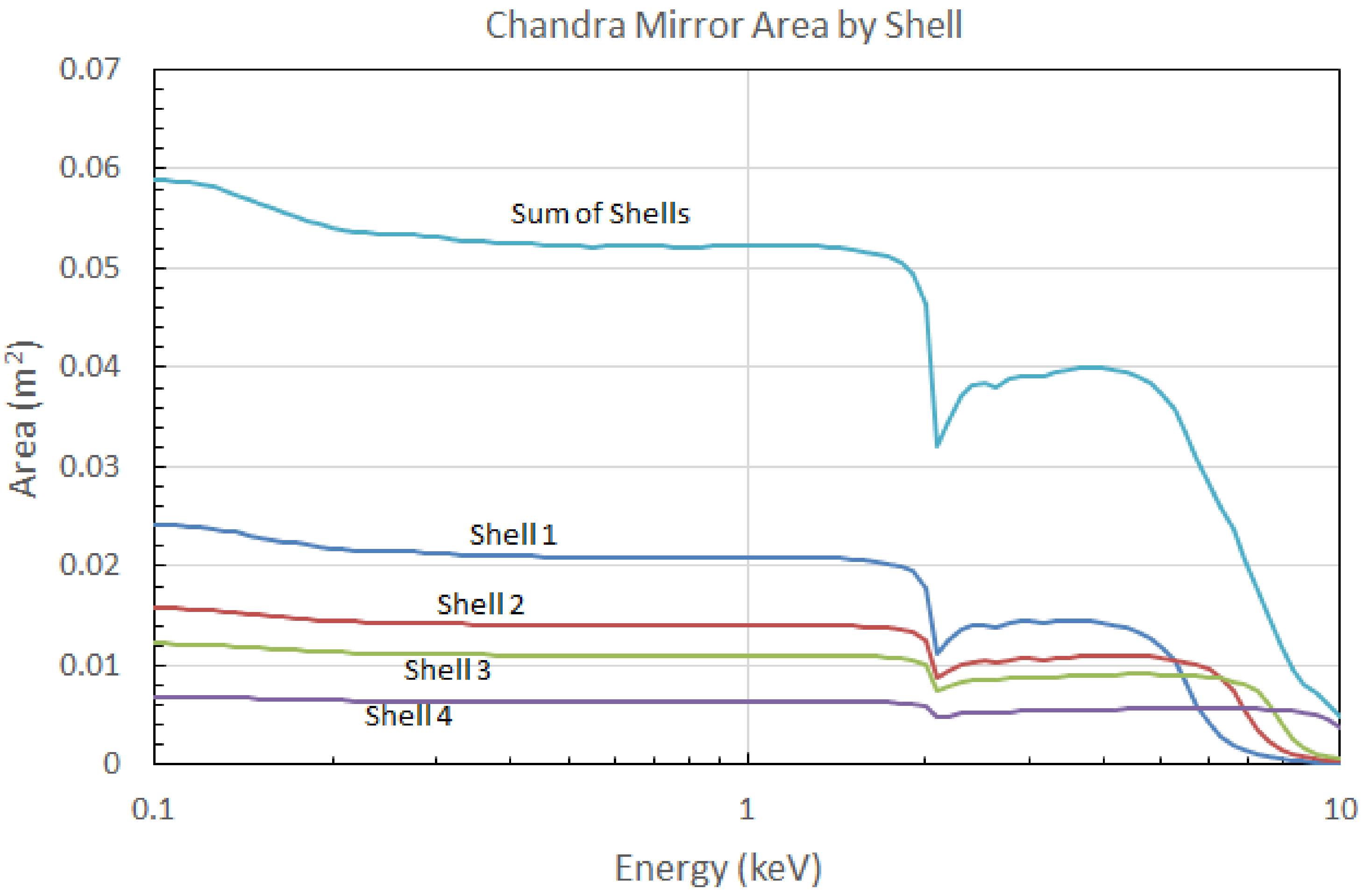
- The critical angle is the angle of total external reflection of x rays
- Mirror reflectivity drops above cutoff energy
- Critical angle $\alpha_c = \sqrt{2\delta(E)}$ scales like $1/E_c$ where E_c is cutoff energy



Mirror Reflectivity Drops above Cutoff Energy

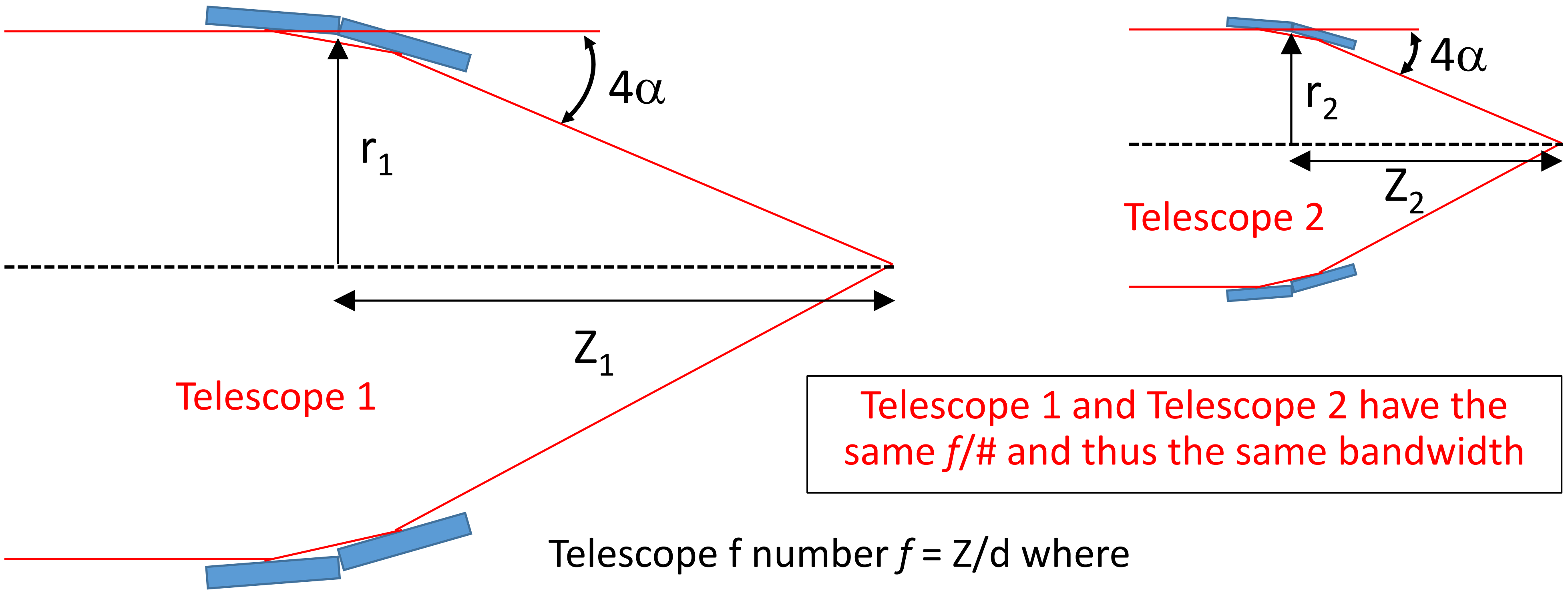


But Telescope Area Tapers Off Gradually Due to Inner Mirrors



Inner mirrors have higher cutoff energy and thus better high-energy response – but they have less area

Telescope *f*/# Determines Bandwidth



Telescope 1 and Telescope 2 have the same *f*/# and thus the same bandwidth

Telescope f number *f* = Z/d where

Z = focal length and d = 2r = mirror diameter.

Note $\tan(4\alpha) = r/Z = 1/2f$, where α is mirror graze angle, so

$$\alpha = \frac{1}{4}tan^{-1}\left(\frac{1}{2f}\right) \rightarrow$$ same for both telescopes!

Solve for Telescope f Number

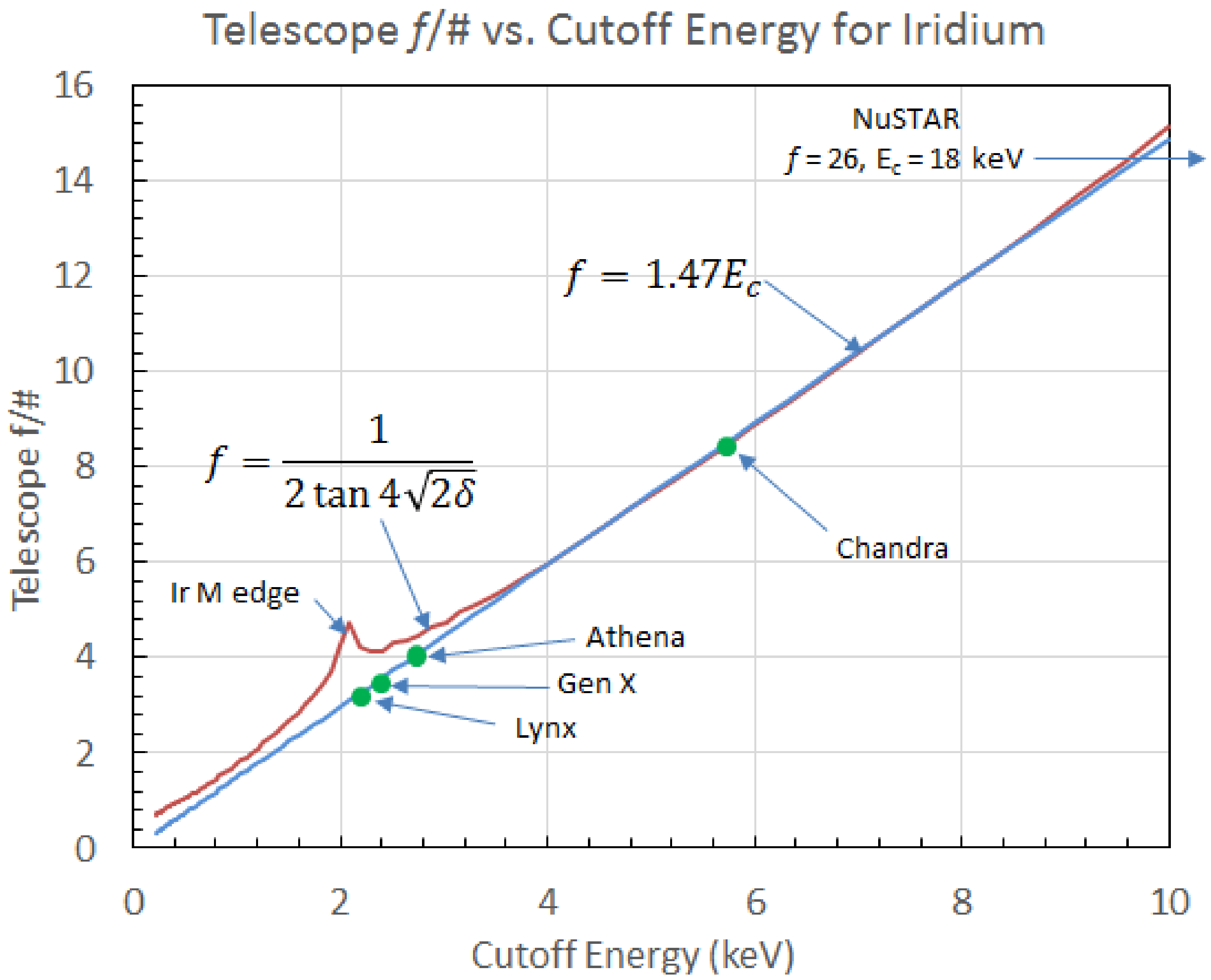
$\alpha = \frac{1}{4} tan^{-1}\left(\frac{1}{2f}\right) \approx \frac{1}{2f}$ from geometry

$\alpha_c = \sqrt{2\delta(E)} \approx c/E$ from physics

Combine these to obtain

$$f = \frac{1}{2\tan 4\sqrt{2\delta}} \approx \frac{1}{2\tan\left(4\frac{c}{E_c}\right)} \approx \frac{E_c}{8c}$$

where c is a constant and E_c is the cutoff energy



Step 2: Telescope Collecting Area

Optimize Mirror Collecting Area

Once you have decided on the telescope band width, focal length is the only free parameter left that controls mirror collecting area

→ Increase focal length until you run out of money ←

- On the plus side: Mirror area increases as square of focal length
- On the minus side: Mirror cost increases as square of focal length

Step 3: Telescope Resolution

Fundamental Contributors to Telescope Resolution

- Wolter-Schwarzschild geometry
- Diffraction
- Build errors (assumed to be zero)
- Scattering (assumed negligible)

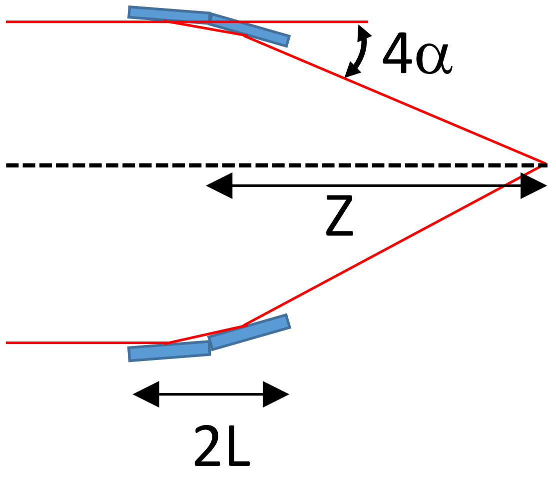
Wolter-Schwarzschild Mirror Parametrization

RMS geometry blur circle radius (radians):

$$\sigma_G = 0.270\,\frac{\tan^2\theta}{\tan\alpha}\frac{L}{Z}$$

RMS diffraction circle radius (radians):

$$\sigma_D = \frac{\lambda}{2L\tan\alpha}$$



Where:

- θ is field of view (FOV) radius
- α is mirror graze angle (determined by E_c)
- L is mirror length (P only, total length is 2L)
- Z is mirror focal length
- λ is x-ray wavelength

Speybroeck and Chase, Applied Optics, Vol. 11, No. 2, p. 440 (1972)
Chase and Speybroeck, Applied Optics, Vol. 12, No 5, p. 1042 (1973)
Harvey, J. X-ray Science and Tech., Vol. 3, p. 68 (1991)

Mirror Top-Level Error Budget

- Geometry term σ_G
- Diffraction term σ_D
- Engineering term σ_E (assumed to be zero)

$$HPD = 2\sqrt{\sigma_G^2 + \sigma_D^2 + \sigma_E^2}$$

$$HPD = 2\sqrt{\left(0.270\,\frac{\tan^2\theta}{\tan\alpha}\frac{L}{Z}\right)^2 + \left(\frac{\lambda}{2L\tan\alpha}\right)^2 + \sigma_E^2}$$

Optimize Mirror Resolution for Given FOV

- Recall α is fixed by telescope bandwidth requirement
  - $\alpha \approx 0.084/E_c$
- Recall focal length Z is fixed by choice of mirror collecting area
- Choose desired FOV angle θ
- Choose target energy E to optimize telescope resolution (note λ = hc/E)

Find optimum mirror length L:

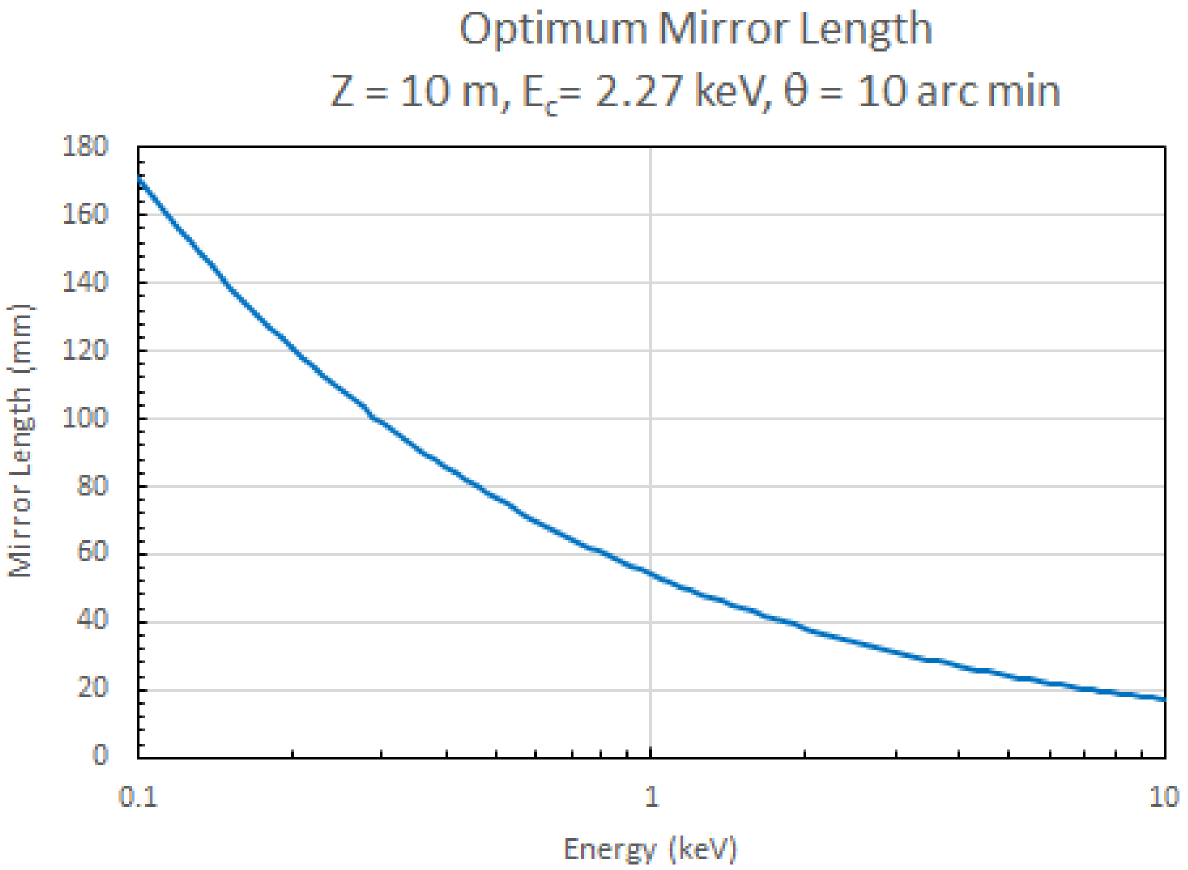
$$\frac{dHPD}{dL} = 0 \qquad \Rightarrow \qquad L = \frac{\sqrt{2\lambda Z}}{\tan\theta}$$

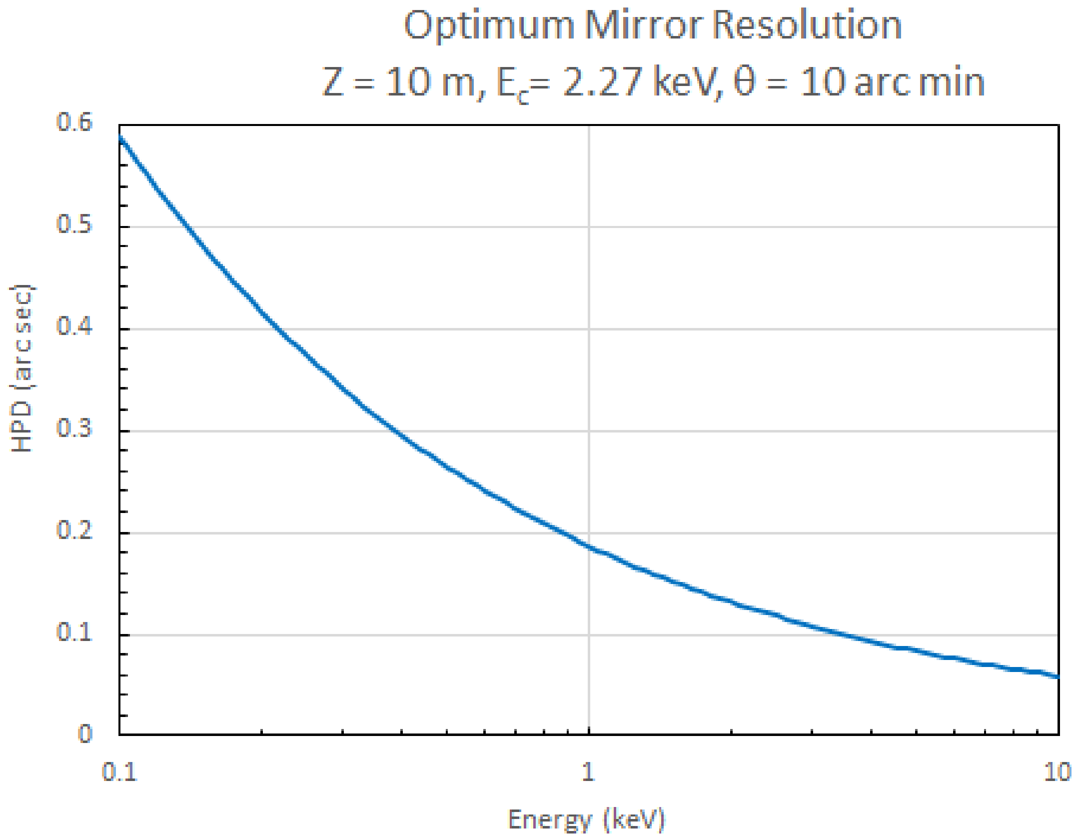
$$HPD_{opt} = 2\sqrt{0.27\frac{\lambda}{Z}\frac{\tan^2\theta}{\tan^2\alpha}} \approx 12.4E_c\tan\theta\sqrt{\frac{\lambda}{Z}}$$

This is the best possible resolution that physics allows
at the target wavelength, FOV, focal length (i.e., collecting area) and bandwidth

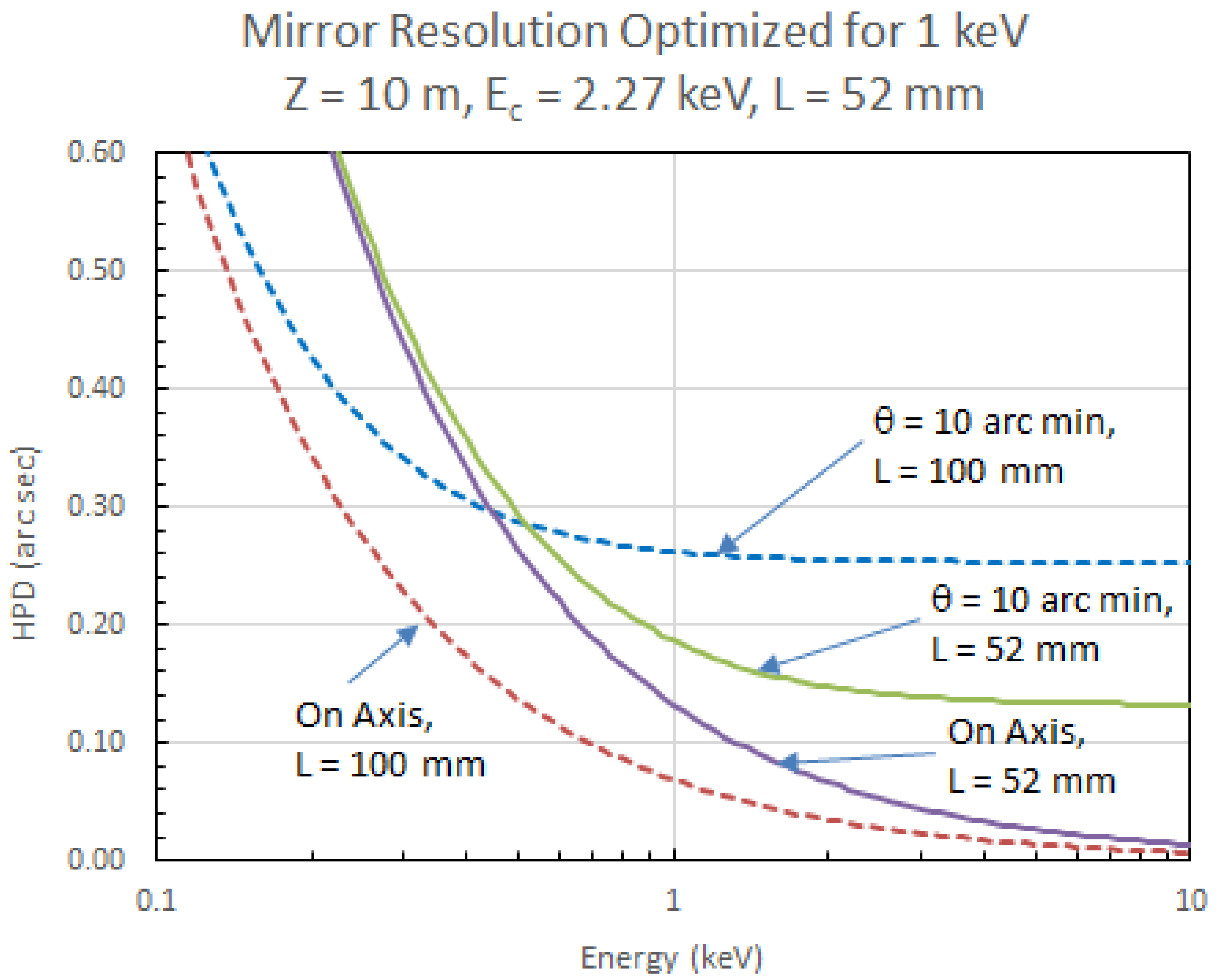
Best Possible Mirror for Nominal Lynx Geometry

Each energy represents a mirror optimized for a specific energy, focal length, energy cutoff and FOV





Nominal Lynx Optimized for 1 keV

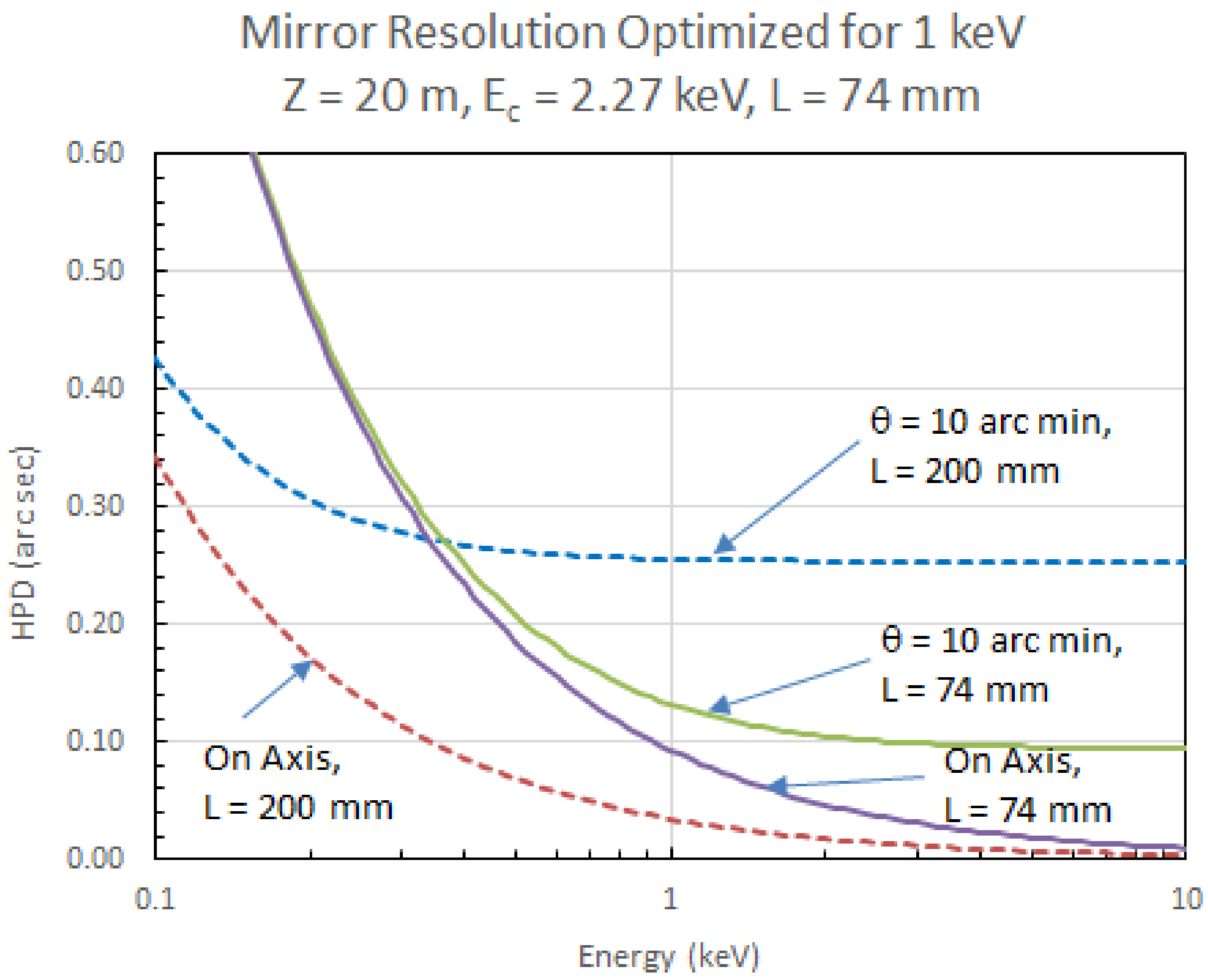


Take away:

- You can make on-axis resolution as small as desired by increasing mirror length, but at the expense of FOV
- Resolution of <0.1 arc sec can only be achieved at the expense of FOV
- A wide-field design (10 arc min) struggles to achieve 0.2 arc second
- Practical mirror lengths (>100 mm) limit wide-field designs to > 0.25 arc sec
- These are physics limits and do not consider the very real engineering challenges

“Super” Lynx Optimized for 1 keV

- Can we achieve Weisskopf’s “dream telescope?”
- Of course on-axis resolution can be as small as desired by increasing mirror length, but only at the expense of FOV
- Optimum HPD scales like $E_c \tan\theta \sqrt{\frac{\lambda}{Z}}$
- So the only lever we have is focal length Z, and then resolution improves only as $\sqrt{1/Z}$ ☹



Summary

- Geometry and physics impose fundamental constraints on telescope performance
- Recommend that mission science considerations start from bandwidth, collecting area, and resolution/FOV considerations, in that order
- In the deep sub-0.5 arc sec domain, diffraction must be seriously considered in order to optimize telescope performance
- FOV considerations are critical in order to optimize telescope performance
- More work is needed to fully understand the trade offs (e.g., ray tracing)
- Room needs to be left in the error budget for the poor engineers!